Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Level

Monday 28 January 2013 – Morning

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M3), the paper reference (6679), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. A particle P is moving along the positive x-axis. When the displacement of P from the origin is x metres, the velocity of P is v m s^{-1} and the acceleration of P is $9x$ m s^{-2}.

When $x = 2$, $v = 6$.

Show that $v^2 = 9x^2$.

(4)

2.

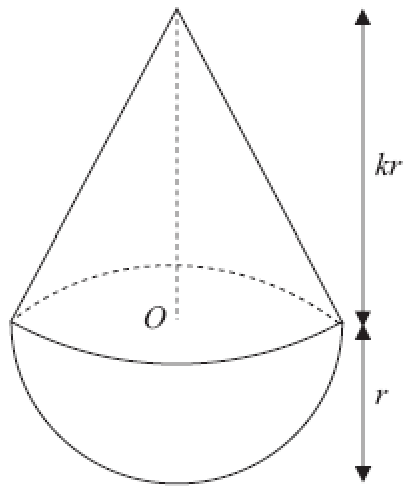

Figure 1

A uniform solid consists of a right circular cone of radius r and height kr, where $k > \sqrt{3}$, fixed to a hemisphere of radius r. The centre of the plane face of the hemisphere is O and this plane face coincides with the base of the cone, as shown in Figure 1.

(*a*) Show that the distance of the centre of mass of the solid from O is

$$\frac{(k^2 - 3)r}{4(k + 2)}.$$

(5)

The point A lies on the circumference of the base of the cone. The solid is suspended by a string attached at A and hangs freely in equilibrium. The angle between AO and the vertical is θ, where $\tan\theta = \frac{11}{14}$.

(*b*) Find the value of k.

(4)

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3. A particle P of mass 0.6 kg is moving along the x-axis in the positive direction. At time $t = 0$, P passes through the origin O with speed 15 m s^{-1}. At time t seconds the distance OP is x metres, the speed of P is v m s^{-1} and the resultant force acting on P has magnitude $\frac{12}{(t+2)^2}$ newtons. The resultant force is directed towards O.

(a) Show that $v = 5\left(\frac{4}{t+2} + 1\right)$.

(5)

(b) Find the value of x when $t = 5$.

(5)

4.

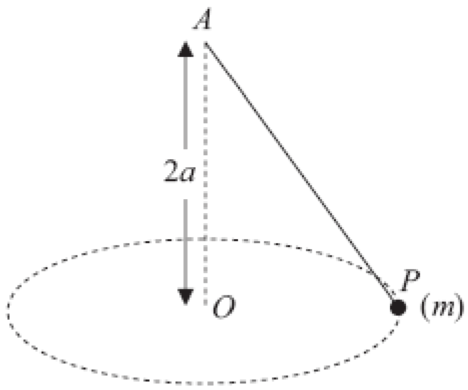

Figure 2

A particle P of mass m is attached to one end of a light elastic string, of natural length $2a$ and modulus of elasticity $6mg$. The other end of the string is attached to a fixed point A. The particle moves with constant speed v in a horizontal circle with centre O, where O is vertically below A and $OA = 2a$, as shown in Figure 2.

(a) Show that the extension in the string is $\frac{2}{5}a$.

(6)

(b) Find v^2 in terms of a and g.

(5)

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5. A particle P is moving in a straight line with simple harmonic motion on a smooth horizontal floor. The particle comes to instantaneous rest at points A and B where AB is 0.5 m. The mid-point of AB is O. The mid-point of OA is C. The mid-point of OB is D. The particle takes 0.2 s to travel directly from C to D. At time $t = 0$, P is moving through O towards A.

(*a*) Show that the period of the motion is $\frac{6}{5}$ s.

(5)

(*b*) Find the distance of P from B when $t = 2$ s.

(3)

(*c*) Find the maximum magnitude of the acceleration of P.

(2)

(*d*) Find the maximum speed of P.

(2)

6.

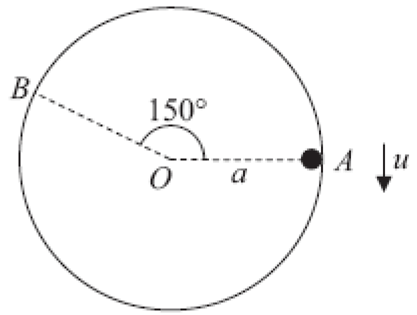

Figure 3

A smooth hollow cylinder of internal radius a is fixed with its axis horizontal. A particle P moves on the inner surface of the cylinder in a vertical circle with radius a and centre O, where O lies on the axis of the cylinder. The particle is projected vertically downwards with speed u from point A on the circle, where OA is horizontal. The particle first loses contact with the cylinder at the point B, where $\angle AOB = 150°$, as shown in Figure 3. Given that air resistance can be ignored,

(*a*) show that the speed of P at B is $\sqrt{\left(\frac{ag}{2}\right)}$,

(3)

(*b*) find u in terms of a and g.

(4)

After losing contact with the cylinder, P crosses the diameter through A at the point D. At D the velocity of P makes an angle $\theta°$ with the horizontal.

(*c*) Find the value of θ.

(7)

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7. A particle P of mass 1.5 kg is attached to the mid-point of a light elastic string of natural length 0.30 m and modulus of elasticity λ newtons. The ends of the string are attached to two fixed points A and B, where AB is horizontal and $AB = 0.48$ m. Initially P is held at rest at the mid-point, M, of the line AB and the tension in the string is 240 N.

(*a*) Show that $\lambda = 400$.

(3)

The particle is now held at rest at the point C, where C is 0.07 m vertically below M. The particle is released from rest at C.

(*b*) Find the magnitude of the initial acceleration of P.

(6)

(*c*) Find the speed of P as it passes through M.

(6)

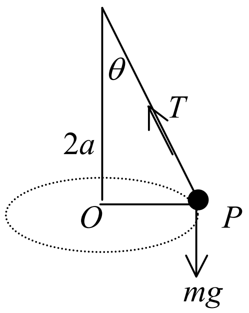
TOTAL FOR PAPER: 75 MARKS

END

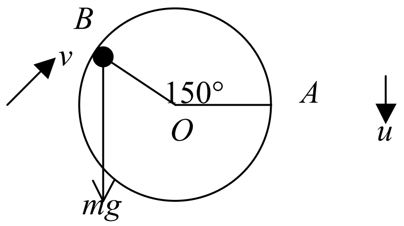
Question Number	Scheme	Marks
1.	$v\dfrac{\mathrm{d}v}{\mathrm{d}x} = 9x$	M1
	$\dfrac{1}{2}v^2 = 9x \quad (+c)$	A1
	$v^2 = 9x^2 + c$	M1dep
	$x = 2 \;\; v = 6 \quad 36 = 9 \times 4 + c \Rightarrow c = 0$	
	$v^2 = 9x^2$	A1

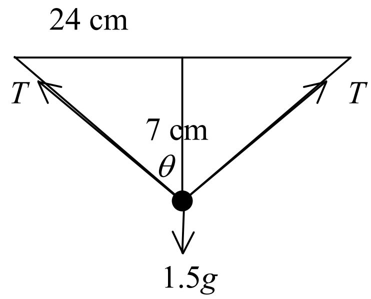
Question Number	Scheme	Marks
2 (a)	Mass: $\frac{2}{3}\pi r^3$ $\frac{1}{3}k\pi r^3$	
	2 k $2+k$	B1
	Dist from O: $-\frac{3}{8}r$ $\frac{1}{4}kr$ $\bar{x}$	B1
	$-\frac{3}{4}r+\frac{k^2r}{4}=\bar{x}(2+k)$	M1A1ft
	$\bar{x}=\frac{(k^2-3)r}{4(k+2)}$ *	A1
(b)		
	$\tan\theta=\frac{(k^2-3)r}{4(k+2)}\div r$	M1A1
	$\frac{(k^2-3)}{4(k+2)}=\frac{11}{14}$	
	$14k^2-42=44k+88$	
	$7k^2-22k-65=0$	
	$(7k+13)(k-5)=0$	
	$k=5$	M1 A1

Question Number	Scheme	Marks
3		
(a)	$0.6a = -\dfrac{12}{(t+2)^2}$	M1
	$0.6\int \mathrm{d}v = -\int \dfrac{12}{(t+2)^2}\,\mathrm{d}t$	
	$0.6v = \dfrac{12}{(t+2)} \quad (+c)$	M1depA1
	$t=0 \quad v=15 \qquad 0.6\times 15 = 6 + c \Rightarrow c = 3$	M1dep
	$0.6v = \dfrac{12}{(t+2)} + 3 \qquad v = \dfrac{20}{(t+2)} + 5 = 5\left(\dfrac{4}{t+2}+1\right)$ *	A1
(b)	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = 5\left(\dfrac{4}{t+2}+1\right)$	M1
	$x = \int 5\left(\dfrac{4}{t+2}+1\right)\mathrm{d}t$	
	$x = 5\left(4\ln(t+2)+t\right) \quad (+c')$	M1 A1
	$t=0,\ x=0 \quad c' = -20\ln 2$	
	$t=5 \quad x = 5(4\ln 7 + 5) - 20\ln 2$ $= 50.05... = 50.1$ or better	M1dep
	or $20\ln\left(\dfrac{7}{2}\right)+25$	A1

Question Number	Scheme	Marks
4		
(a)	R(↑) $T\cos\theta = mg$	M1
	$T \times \frac{2a}{(2a+x)} = mg$	A1
	Hooke's Law: $T = \frac{6mgx}{2a} = \frac{3mgx}{a}$	M1A1
	$\frac{3mgx}{a} \times \frac{2a}{(2a+x)} = mg$	M1dep
	$6x = 2a + x$	
	$x = \frac{2}{5}a$ *	A1
(b)	$T\sin\theta = \frac{mv^2}{r}$	M1 A1
	$3mg \times \frac{2}{5}\sin\theta = \frac{mv^2}{\left(\frac{12a}{5}\right)\sin\theta}$	M1
	$v^2 = \frac{6}{5}g \times \frac{12a}{5}\sin^2\theta$	
	$\sin^2\theta = 1 - \left(\frac{4a^2}{\left(\frac{12a}{5}\right)^2}\right) = \frac{11}{36}$	
	$v^2 = \frac{72ag}{25} \times \frac{11}{36} = \frac{22ag}{25}$	M1 A1

Question Number	Scheme	Marks
5. (a)	$x = a\sin\omega t$	
	$0.125 = 0.25\sin 0.1\omega$	M1 A1
	$\sin 0.1\omega = \dfrac{1}{2}$	
	$0.1\omega = \dfrac{\pi}{6}$	
	$\omega = \dfrac{\pi}{0.6} = \dfrac{10\pi}{6}$	M1 A1
	Period $= \dfrac{2\pi}{\omega} = \dfrac{6}{5} \quad (=1.2)$	A1
(b)	$x = 0.25\sin\dfrac{5}{3}\pi t$	
	$t = 2 \quad x = 0.25\sin\left(2\times\dfrac{5}{3}\pi\right)$	M1
	$x = -0.2165...$	A1
	Dist from $B = 0.25 + x = 0.033$ m	A1 ft
(c)	Max accel $= a\omega^2 = 0.25\times\left(\dfrac{5\pi}{3}\right)^2 = 6.853... = 6.85$	M1A1
(d)	Max speed $a\omega = 0.25\times\left(\dfrac{5\pi}{3}\right) = 1.308... = 1.31$	M1A1

Question Number	Scheme	Marks
6		
(a)	At B $\quad mg\cos 60\ (+R) = \dfrac{mv^2}{a}$	M1A1
	$\dfrac{1}{2}g = \dfrac{v^2}{a} \qquad v = \sqrt{\dfrac{ag}{2}}$ *	A1
(b)	Energy A to B: $\dfrac{1}{2}mu^2 - \dfrac{1}{2}m\left(\dfrac{ag}{2}\right) = mga\sin 30$	M1A1A1
	$u^2 = \dfrac{ag}{2} + 2ag \times \dfrac{1}{2}$	
	$u = \sqrt{\dfrac{3ag}{2}}$	A1
(c)	Horiz speed = $\sqrt{\dfrac{ag}{2}}\cos 60 \quad \left(=\dfrac{1}{2}\sqrt{\dfrac{ag}{2}}\right)$	M1A1
	Initial vert speed = $(-)\sqrt{\dfrac{ag}{2}}\sin 60 \quad \left(=(-)\dfrac{1}{2}\sqrt{\dfrac{3ag}{2}}\right)$	M1
	$v^2 = \dfrac{1}{4}\times\dfrac{3ag}{2} + 2g\times\dfrac{a}{2}$	M1A1
	$v^2 = \dfrac{11ag}{8}$	
	$\tan\theta = \dfrac{\text{vert}}{\text{horiz}} = \sqrt{\dfrac{11ag}{8}\times\dfrac{8}{ag}} = \sqrt{11}$	M1
	$\theta = 73.22... = 73$	A1

Question Number	Scheme	Marks
7 (a)	$T=\frac{\lambda x}{l}\Rightarrow 240=\frac{\lambda\times 18}{30}$	M1A1
	$\lambda=400$	A1
(b)		
	Extension = 10 cm or 20 cm (used in (b) or (c))	B1
	$T=\frac{400\times 10}{15}=\left(\frac{800}{3}\right)$	M1A1ft
	$\text{R}(\uparrow)\quad 2T\cos\theta-1.5g=(\pm)1.5a$	M1A1
	$\frac{1600}{3}\times\frac{7}{25}-1.5\times 9.8=(\pm)1.5a$	
	$a=89.75...\quad a=90\ \text{m s}^{-2}$ or 89.8 (positive)	A1
(c)	$\text{E.P.E.}=\frac{1}{2}\times 400\times\frac{0.2^2}{0.3}$	B1ft (any correct EPE)
	$1.5g\times 0.07+\frac{1}{2}\times 1.5v^2=200\times\frac{0.2^2}{0.3}-\frac{200\times 0.18^2}{0.3}$	M1A1A1
	$v^2=\frac{1}{0.75}\left(200\times\frac{0.2^2}{0.3}-\frac{200\times 0.18^2}{0.3}-1.5g\times 0.07\right)$	M1dep
	$v=2.32...=2.3\ \text{m s}^{-1}$	A1